Example 12 **Self Tutor**

Determine the validity of the following argument:

If a triangle has three sides, then $2+4=7$.
$2+4=7$
Hence, a triangle has three sides.

We have p: A triangle has three sides and q: $2+4=7$

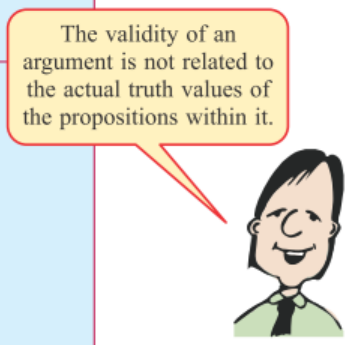
The argument is:

$$\begin{array}{r} p \Rightarrow q \\ q \\ \hline p \end{array} \quad \left.\begin{array}{r} \\ \\ \end{array}\right\} \text{premise} \quad \left.\begin{array}{r} \\ \end{array}\right\} \text{conclusion}$$

We can write this in logical form as $(p \Rightarrow q) \wedge q \Rightarrow p$.

p	q	$p \Rightarrow q$	$(p \Rightarrow q) \wedge q$	$(p \Rightarrow q) \wedge q \Rightarrow p$
T	T	T	T	T
T	F	F	F	T
F	T	T	T	F
F	F	T	F	T

Since we do **not** have a tautology, the argument is not valid.



Example 13 Self Tutor

Determine the validity of the following argument:

If x is a natural number, then x is an integer.
If x is an integer, then x is rational.
Therefore, if x is a natural number, then x is rational.

We have p: x is a natural number, q: x is an integer, and r: x is rational.

The argument is written as
$$\begin{array}{l} p \Rightarrow q \\ \underline{q \Rightarrow r} \\ p \Rightarrow r \end{array}$$

We can write this in logical form as $(p \Rightarrow q) \wedge (q \Rightarrow r) \Rightarrow (p \Rightarrow r)$.

p	q	r	$p \Rightarrow q$	$q \Rightarrow r$	$(p \Rightarrow q) \wedge (q \Rightarrow r)$	$p \Rightarrow r$	$(p \Rightarrow q) \wedge (q \Rightarrow r) \Rightarrow (p \Rightarrow r)$
T	T	T	T	T	T	T	T
T	T	F	T	F	F	F	T
T	F	T	F	T	F	T	T
T	F	F	F	T	F	F	T
F	T	T	T	T	T	T	T
F	T	F	T	F	F	T	T
F	F	T	T	T	T	T	T
F	F	F	T	T	T	T	T

The logical form of the argument is a tautology, so the argument is valid.