Example 5 Self Tutor

Construct a truth table for $p \vee \neg q$.

We start by listing all the possible combinations for p and q:

p	q	$\neg q$	$p \vee \neg q$
T	T		
T	F		
F	T		
F	F		

We then use the negation rule on the q column to find $\neg q$:

p	q	$\neg q$	$p \vee \neg q$
T	T	F	
T	F	T	
F	T	F	
F	F	T	

Finally, we use the disjunction rule on the p and $\neg q$ columns to find $p \vee \neg q$:

Finally, we use the disjunction rule on the p and $\neg q$ columns to find $p \vee \neg q$:

p	q	$\neg q$	$p \vee \neg q$
T	T	F	T
T	F	T	T
F	T	F	F
F	F	T	T

Example 7 Self Tutor

Show that $(\neg q \wedge p) \wedge (q \vee \neg p)$ is a logical contradiction.

The truth table is:

p	q	$\neg p$	$\neg q$	$(\neg q \wedge p)$	$(q \vee \neg p)$	$(\neg q \wedge p) \wedge (q \vee \neg p)$
T	T	F	F	F	T	F
T	F	F	T	T	F	F
F	T	T	F	F	T	F
F	F	T	T	F	T	F

All the values in the final column are false, so $(\neg q \wedge p) \wedge (q \vee \neg p)$ is a logical contradiction.

Example 8 Self Tutor

Show that $\neg(p \wedge q)$ and $\neg p \vee \neg q$ are logically equivalent.

The truth table for $\neg(p \wedge q)$ is:

p	q	$p \wedge q$	$\neg(p \wedge q)$
T	T	T	F
T	F	F	T
F	T	F	T
F	F	F	T

The truth table for $\neg p \vee \neg q$ is:

p	q	$\neg p$	$\neg q$	$\neg p \vee \neg q$
T	T	F	F	F
T	F	F	T	T
F	T	T	F	T
F	F	T	T	T

Since the truth table columns for $\neg(p \wedge q)$ and $\neg p \vee \neg q$ are identical, $\neg(p \wedge q)$ and $\neg p \vee \neg q$ are logically equivalent.

So, $\neg(p \wedge q) = \neg p \vee \neg q$.

Example 9 Self Tutor

Construct a truth table for the compound proposition $(p \vee q) \wedge r$.

To find $(p \vee q) \wedge r$, we first find $p \vee q$.
We then find the conjunction of $p \vee q$ and r.

p	q	r	$p \vee q$	$(p \vee q) \wedge r$
T	T	T	T	T
T	T	F	T	F
T	F	T	T	T
T	F	F	T	F
F	T	T	T	T
F	T	F	T	F
F	F	T	F	F
F	F	F	F	F