

**Example 1****Self Tutor**

$M = \{2, 3, 5, 7, 8, 9\}$  and  $N = \{3, 4, 6, 9, 10\}$

- a** True or false?    **i**  $4 \in M$         **ii**  $6 \notin M$   
**b** List the sets:        **i**  $M \cap N$         **ii**  $M \cup N$   
**c** Is    **i**  $M \subseteq N$         **ii**  $\{9, 6, 3\} \subseteq N$ ?

- a**    **i** 4 is not an element of  $M$ , so  $4 \in M$  is false.  
      **ii** 6 is not an element of  $M$ , so  $6 \notin M$  is true.  
**b**    **i**  $M \cap N = \{3, 9\}$  since 3 and 9 are elements of both sets.  
      **ii** Every element which is in either  $M$  or  $N$  is in the union of  $M$  and  $N$ .  
       $\therefore M \cup N = \{2, 3, 4, 5, 6, 7, 8, 9, 10\}$   
**c**    **i** No. Not every element of  $M$  is an element of  $N$ .  
      **ii** Yes, as 9, 6, and 3 are also in  $N$ .

**Example 2****Self Tutor**

Suppose  $A = \{x \mid 3 < x \leq 10, x \in \mathbb{Z}\}$ .

- a** Write down the meaning of the set builder notation.  
**b** List the elements of set  $A$ .                      **c** Find  $n(A)$ .
- a** The set of all  $x$  such that  $x$  is an integer between 3 and 10, including 10.  
**b**  $A = \{4, 5, 6, 7, 8, 9, 10\}$                       **c** There are 7 elements, so  $n(A) = 7$ .

**Example 3** **Self Tutor**Find  $C'$  given that:

- a**  $U = \{\text{all positive integers}\}$  and  $C = \{\text{all even integers}\}$   
**b**  $C = \{x \mid x \geq 2, x \in \mathbb{Z}\}$  and  $U = \mathbb{Z}$

- a**  $C' = \{\text{all odd integers}\}$                       **b**  $C' = \{x \mid x \leq 1, x \in \mathbb{Z}\}$

**Example 4** **Self Tutor**Suppose  $U = \{x \mid -5 \leq x \leq 5, x \in \mathbb{Z}\}$ ,  $A = \{x \mid 1 \leq x \leq 4, x \in \mathbb{Z}\}$ , and  $B = \{x \mid -3 \leq x < 2, x \in \mathbb{Z}\}$ . List the elements of these sets:

- a**  $A$                       **b**  $B$                       **c**  $A'$                       **d**  $B'$   
**e**  $A \cap B$               **f**  $A \cup B$               **g**  $A' \cap B$               **h**  $A' \cup B'$

- a**  $A = \{1, 2, 3, 4\}$                       **b**  $B = \{-3, -2, -1, 0, 1\}$   
**c**  $A' = \{-5, -4, -3, -2, -1, 0, 5\}$                       **d**  $B' = \{-5, -4, 2, 3, 4, 5\}$   
**e**  $A \cap B = \{1\}$                       **f**  $A \cup B = \{-3, -2, -1, 0, 1, 2, 3, 4\}$   
**g**  $A' \cap B = \{-3, -2, -1, 0\}$   
**h**  $A' \cup B' = \{-5, -4, -3, -2, -1, 0, 2, 3, 4, 5\}$

**Example 5** **Self Tutor**Suppose  $U = \{\text{positive integers}\}$ ,  $P = \{\text{multiples of 4 less than 50}\}$ , and  $Q = \{\text{multiples of 6 less than 50}\}$ .

- a** List  $P$  and  $Q$ .                      **b** Find  $P \cap Q$ .                      **c** Find  $P \cup Q$ .  
**d** Verify that  $n(P \cup Q) = n(P) + n(Q) - n(P \cap Q)$ .

- a**  $P = \{4, 8, 12, 16, 20, 24, 28, 32, 36, 40, 44, 48\}$   
 $Q = \{6, 12, 18, 24, 30, 36, 42, 48\}$   
**b**  $P \cap Q = \{12, 24, 36, 48\}$   
**c**  $P \cup Q = \{4, 6, 8, 12, 16, 18, 20, 24, 28, 30, 32, 36, 40, 42, 44, 48\}$   
**d**  $n(P \cup Q) = 16$  and  $n(P) + n(Q) - n(P \cap Q) = 12 + 8 - 4 = 16$   
 So,  $n(P \cup Q) = n(P) + n(Q) - n(P \cap Q)$  is verified.