Example 12 Self Tutor

Find the sum of $4 + 7 + 10 + 13 + \ldots.$ to 50 terms.

The series is arithmetic with $u_1 = 4$, $d = 3$ and $n = 50$.

Now $S_n = \frac{n}{2}(2u_1 + (n-1)d)$

$\therefore \; S_{50} = \frac{50}{2}(2 \times 4 + 49 \times 3)$

$= 3875$

Example 13 Self Tutor

Find the sum of $-6 + 1 + 8 + 15 + \ldots. + 141$.

The series is arithmetic with $u_1 = -6$, $d = 7$ and $u_n = 141$.

First we need to find n.

Now $u_n = 141$

$\therefore \; u_1 + (n-1)d = 141$

$\therefore \; -6 + 7(n-1) = 141$

$\therefore \; 7(n-1) = 147$

$\therefore \; n - 1 = 21$

$\therefore \; n = 22$

Using $S_n = \frac{n}{2}(u_1 + u_n)$,

$S_{22} = \frac{22}{2}(-6 + 141)$

$= 11 \times 135$

$= 1485$

Example 14 Self Tutor

An arithmetic sequence has first term 8 and common difference 2. The sum of the terms of the sequence is 170. Find the number of terms in the sequence.

The sequence is arithmetic with $u_1 = 8$ and $d = 2$.

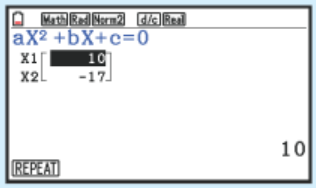
Now $S_n = 170$, so $\frac{n}{2}(2u_1 + (n-1)d) = 170$

$\therefore \frac{n}{2}(16 + 2(n-1)) = 170$

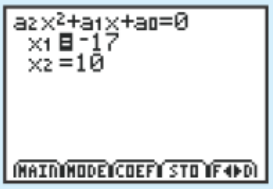
$\therefore 8n + n(n-1) = 170$

$\therefore n^2 + 7n - 170 = 0$

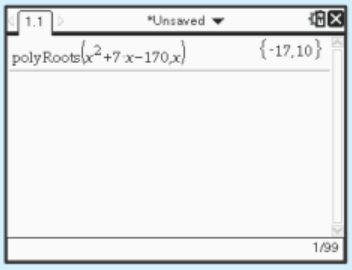
Casio fx-CG20



TI-84 Plus



TI-*n*spire



$\therefore n = -17$ or 10 {technology}

But $n > 0$, so $n = 10$

$\therefore$ there are 10 terms in the sequence.

Example 16 Self Tutor

Find a formula for S_n for the first n terms of $9 - 3 + 1 - \frac{1}{3} + ...$

The series is geometric with $u_1 = 9$ and $r = -\frac{1}{3}$

$$S_n = \frac{u_1(1-r^n)}{1-r} = \frac{9(1-(-\frac{1}{3})^n)}{\frac{4}{3}}$$

$\therefore S_n = \frac{27}{4}(1-(-\frac{1}{3})^n)$

Example 17 **Self Tutor**

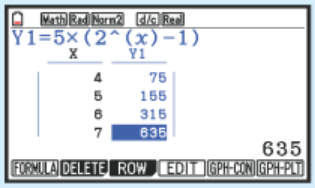
A geometric sequence has first term 5 and common ratio 2. The sum of the first n terms of the sequence is 635. Find n.

The sequence is geometric with $u_1 = 5$ and $r = 2$.

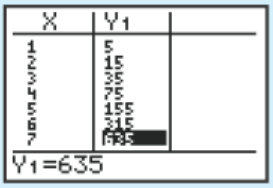
$$\therefore \ S_n = \frac{u_1(r^n - 1)}{r - 1}$$
$$= \frac{5(2^n - 1)}{2 - 1}$$
$$= 5(2^n - 1)$$

To find n such that $S_n = 635$, we use a table of values with $Y_1 = 5 \times (2\hat{}\,X - 1)$:

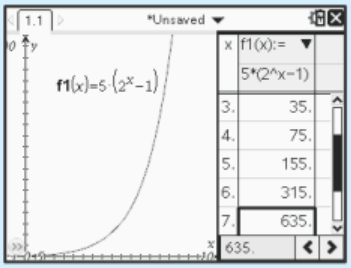
Casio fx-CG20



TI-84 Plus



TI-*n*spire



So, $S_7 = 635$ and $\therefore \ n = 7$.