

Example 1**Self Tutor**Simplify: **a** $7^4 \times 7^5$ **b** $p^6 \times p^2$

$$\begin{aligned}\mathbf{a} \quad & 7^4 \times 7^5 \\&= 7^{4+5} \\&= 7^9\end{aligned}\quad \begin{aligned}\mathbf{b} \quad & p^6 \times p^2 \\&= p^{6+2} \\&= p^8\end{aligned}$$

These have the form
 $a^m \times a^n = a^{m+n}$

**Example 2****Self Tutor**

Simplify:

$$\begin{aligned}\mathbf{a} \quad & \frac{5^6}{5^3} \\&= 5^{6-3} \\&= 5^3\end{aligned}\quad \begin{aligned}\mathbf{b} \quad & \frac{x^{11}}{x^6} \\&= x^{11-6} \\&= x^5\end{aligned}$$

These have the form
 $\frac{a^m}{a^n} = a^{m-n}$

**Example 3****Self Tutor**

Simplify:

$$\begin{aligned}\mathbf{a} \quad & (3^5)^2 \\&= 3^{5 \times 2} \\&= 3^{10}\end{aligned}\quad \begin{aligned}\mathbf{b} \quad & (x^3)^k \\&= x^{3 \times k}\end{aligned}$$

These have the form
 $(a^m)^n = a^{m \times n}$

**Example 4****Self Tutor**

Write as powers of 2:

$$\begin{array}{lllll} \mathbf{a} \quad 16 & \mathbf{b} \quad \frac{1}{16} & \mathbf{c} \quad 1 & \mathbf{d} \quad 4 \times 2^n & \mathbf{e} \quad \frac{2^m}{8} \\[1ex] & = 2 \times 2 \times 2 \times 2 & & & \\ & = 2^4 & & & \\ & & = \frac{1}{2^4} & & \\ & & = 2^{-4} & & \\[1ex] & & & = 2^0 & \\ & & & = 2^{2+n} & \\ & & & = 2^{2+n} & \\ & & & & = \frac{2^m}{2^3} \\ & & & & = 2^{m-3} \end{array}$$

Example 5**Self Tutor**

Express in simplest form with a prime number base:

a 9^4

b $\frac{3^x}{9y}$

c 25^x

$$\begin{aligned} \text{a } 9^4 &= (3^2)^4 \\ &= 3^{2 \times 4} \\ &= 3^8 \end{aligned}$$

$$\begin{aligned} \text{b } \frac{3^x}{9y} &= \frac{3^x}{(3^2)y} \\ &= \frac{3^x}{3^2y} \\ &= 3^{x-2}y \end{aligned}$$

$$\begin{aligned} \text{c } 25^x &= (5^2)^x \\ &= 5^{2x} \end{aligned}$$

Decide first what the prime number base should be.

**Example 6****Self Tutor**

Write without brackets:

a $(3x)^3$

b $\left(\frac{x}{y}\right)^4$

$$\begin{aligned} \text{a } (3x)^3 &= 3^3 \times x^3 \\ &= 27x^3 \end{aligned}$$

$$\begin{aligned} \text{b } \left(\frac{x}{y}\right)^4 &= \frac{x^4}{y^4} \end{aligned}$$

These have the form $(ab)^n = a^n b^n$ or $\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$

**Example 7****Self Tutor**

Simplify, giving answers in simplest rational form:

a 7^0

b 3^{-2}

c $3^0 - 3^{-1}$

d $\left(\frac{5}{3}\right)^{-2}$

$$\begin{aligned} \text{a } 7^0 &= 1 \\ &= 1 \end{aligned}$$

$$\begin{aligned} \text{b } 3^{-2} &= \frac{1}{3^2} \\ &= \frac{1}{9} \end{aligned}$$

$$\begin{aligned} \text{c } 3^0 - 3^{-1} &= 1 - \frac{1}{3} \\ &= \frac{2}{3} \end{aligned}$$

$$\begin{aligned} \text{d } \left(\frac{5}{3}\right)^{-2} &= \left(\frac{3}{5}\right)^2 \\ &= \frac{9}{25} \end{aligned}$$

Notice that

$$\left(\frac{a}{b}\right)^{-2} = \left(\frac{b}{a}\right)^2$$



Example 8**Self Tutor**

Write without negative exponents:

a $3x^{-1}$

b $(3x)^{-1}$

c $\left(\frac{3}{x}\right)^{-2}$

a $3x^{-1}$

$= \frac{3}{x}$

b $(3x)^{-1}$

$= \frac{1}{3x}$

c $\left(\frac{3}{x}\right)^{-2}$

$$\begin{aligned} &= \left(\frac{x}{3}\right)^2 \\ &= \frac{x^2}{3^2} \\ &= \frac{x^2}{9} \end{aligned}$$

$$a^{-n} = \frac{1}{a^n}$$

**Example 10****Self Tutor**

Simplify using the laws of exponents:

a $4x^3 \times 2x^6$

b $\frac{15t^7}{3t^5}$

c $\frac{k^2 \times k^6}{(k^3)^2}$

a $4x^3 \times 2x^6$

$$\begin{aligned} &= 4 \times 2 \times x^3 \times x^6 \\ &= 8 \times x^{3+6} \\ &= 8x^9 \end{aligned}$$

b $\frac{15t^7}{3t^5}$

$$\begin{aligned} &= \frac{15}{3} \times t^{7-5} \\ &= 5t^2 \end{aligned}$$

c $\frac{k^2 \times k^6}{(k^3)^2}$

$$\begin{aligned} &= \frac{k^{2+6}}{k^{3 \times 2}} \\ &= \frac{k^8}{k^6} \\ &= k^2 \end{aligned}$$

Example 13**Self Tutor**

Write in non-fractional form:

a $\frac{x^2 + 3x + 2}{x}$

b $\frac{x^3 + 5x - 3}{x^2}$

c $\frac{2x^5 + x^2 + 3x}{x^{-2}}$

a $\frac{x^2 + 3x + 2}{x}$

$$\begin{aligned} &= \frac{x^2}{x} + \frac{3x}{x} + \frac{2}{x} \\ &= x + 3 + 2x^{-1} \end{aligned}$$

b $\frac{x^3 + 5x - 3}{x^2}$

$$\begin{aligned} &= \frac{x^3}{x^2} + \frac{5x}{x^2} - \frac{3}{x^2} \\ &= x + 5x^{-1} - 3x^{-2} \end{aligned}$$

c $\frac{2x^5 + x^2 + 3x}{x^{-2}}$

$$\begin{aligned} &= \frac{2x^5}{x^{-2}} + \frac{x^2}{x^{-2}} + \frac{3x}{x^{-2}} \\ &= 2x^{5-(-2)} + x^{2-(-2)} \\ &\quad + 3x^{1-(-2)} \\ &= 2x^7 + x^4 + 3x^3 \end{aligned}$$