Example 1

Self Tutor

Simplify: **a** $7^4 \times 7^5$ **b** $p^6 \times p^2$

a $\quad 7^4 \times 7^5$ **b** $\quad p^6 \times p^2$
$\quad\quad = 7^{4+5}$ $\quad\quad = p^{6+2}$
$\quad\quad = 7^9$ $\quad\quad = p^8$

These have the form
$a^m \times a^n = a^{m+n}$

Example 2

Self Tutor

Simplify:

a $\dfrac{5^6}{5^3}$ **b** $\dfrac{x^{11}}{x^6}$

a $\quad \dfrac{5^6}{5^3}$ **b** $\quad \dfrac{x^{11}}{x^6}$
$\quad\quad = 5^{6-3}$ $\quad\quad = x^{11-6}$
$\quad\quad = 5^3$ $\quad\quad = x^5$

These have the form
$\dfrac{a^m}{a^n} = a^{m-n}$

Example 3

Self Tutor

Simplify:

a $(3^5)^2$ **b** $(x^3)^k$

a $\quad (3^5)^2$ **b** $\quad (x^3)^k$
$\quad\quad = 3^{5 \times 2}$ $\quad\quad = x^{3 \times k}$
$\quad\quad = 3^{10}$ $\quad\quad = x^{3k}$

These have the form
$(a^m)^n = a^{m \times n}$

Example 4

Self Tutor

Write as powers of 2:

a 16 **b** $\frac{1}{16}$ **c** 1 **d** 4×2^n **e** $\dfrac{2^m}{8}$

a $\quad 16$ **b** $\quad \dfrac{1}{16}$ **c** $\quad 1$ **d** $\quad 4 \times 2^n$ **e** $\quad \dfrac{2^m}{8}$
$\quad = 2 \times 2 \times 2 \times 2$ $\quad = \dfrac{1}{2^4}$ $\quad = 2^0$ $\quad = 2^2 \times 2^n$ $\quad = \dfrac{2^m}{2^3}$
$\quad = 2^4$ $\quad = 2^{-4}$ $\quad = 2^{2+n}$ $\quad = 2^{m-3}$

Example 5 🔊 **Self Tutor**

Express in simplest form with a prime number base:

a 9^4 **b** $\dfrac{3^x}{9^y}$ **c** 25^x

a 9^4
$= (3^2)^4$
$= 3^{2\times4}$
$= 3^8$

b $\dfrac{3^x}{9^y}$
$= \dfrac{3^x}{(3^2)^y}$
$= \dfrac{3^x}{3^{2y}}$
$= 3^{x-2y}$

c 25^x
$= (5^2)^x$
$= 5^{2x}$

Example 6 🔊 **Self Tutor**

Write without brackets:

a $(3x)^3$ **b** $\left(\dfrac{x}{y}\right)^4$

These have the form $(ab)^n = a^n b^n$ or $\left(\dfrac{a}{b}\right)^n = \dfrac{a^n}{b^n}$

a $(3x)^3$
$= 3^3 \times x^3$
$= 27x^3$

b $\left(\dfrac{x}{y}\right)^4$
$= \dfrac{x^4}{y^4}$

Example 7 🔊 **Self Tutor**

Simplify, giving answers in simplest rational form:

a 7^0 **b** 3^{-2} **c** $3^0 - 3^{-1}$ **d** $\left(\dfrac{5}{3}\right)^{-2}$

Notice that $\left(\dfrac{a}{b}\right)^{-2} = \left(\dfrac{b}{a}\right)^2$

a 7^0
$= 1$

b 3^{-2}
$= \dfrac{1}{3^2}$
$= \dfrac{1}{9}$

c $3^0 - 3^{-1}$
$= 1 - \dfrac{1}{3}$
$= \dfrac{2}{3}$

d $\left(\dfrac{5}{3}\right)^{-2}$
$= \left(\dfrac{3}{5}\right)^2$
$= \dfrac{9}{25}$

Example 8 🔊 **Self Tutor**

Write without negative exponents:

a $3x^{-1}$ **b** $(3x)^{-1}$ **c** $\left(\dfrac{3}{x}\right)^{-2}$

$a^{-n} = \dfrac{1}{a^n}$

a $3x^{-1}$

$= \dfrac{3}{x}$

b $(3x)^{-1}$

$= \dfrac{1}{3x}$

c $\left(\dfrac{3}{x}\right)^{-2}$

$= \left(\dfrac{x}{3}\right)^{2}$

$= \dfrac{x^2}{3^2}$

$= \dfrac{x^2}{9}$

Example 10 🔊 **Self Tutor**

Simplify using the laws of exponents:

a $4x^3 \times 2x^6$ **b** $\dfrac{15t^7}{3t^5}$ **c** $\dfrac{k^2 \times k^6}{(k^3)^2}$

a $4x^3 \times 2x^6$

$= 4 \times 2 \times x^3 \times x^6$

$= 8 \times x^{3+6}$

$= 8x^9$

b $\dfrac{15t^7}{3t^5}$

$= \dfrac{15}{3} \times t^{7-5}$

$= 5t^2$

c $\dfrac{k^2 \times k^6}{(k^3)^2}$

$= \dfrac{k^{2+6}}{k^{3\times 2}}$

$= \dfrac{k^8}{k^6}$

$= k^2$

Example 13 🔊 **Self Tutor**

Write in non-fractional form:

a $\dfrac{x^2 + 3x + 2}{x}$ **b** $\dfrac{x^3 + 5x - 3}{x^2}$ **c** $\dfrac{2x^5 + x^2 + 3x}{x^{-2}}$

a $\dfrac{x^2 + 3x + 2}{x}$

$= \dfrac{x^2}{x} + \dfrac{3x}{x} + \dfrac{2}{x}$

$= x + 3 + 2x^{-1}$

b $\dfrac{x^3 + 5x - 3}{x^2}$

$= \dfrac{x^3}{x^2} + \dfrac{5x}{x^2} - \dfrac{3}{x^2}$

$= x + 5x^{-1} - 3x^{-2}$

c $\dfrac{2x^5 + x^2 + 3x}{x^{-2}}$

$= \dfrac{2x^5}{x^{-2}} + \dfrac{x^2}{x^{-2}} + \dfrac{3x}{x^{-2}}$

$= 2x^{5-(-2)} + x^{2-(-2)}$

$\qquad + 3x^{1-(-2)}$

$= 2x^7 + x^4 + 3x^3$