Example 7

Self Tutor

Consider the sequence $8, 4, 2, 1, \frac{1}{2}, ...$

a Show that the sequence is geometric.

b Find the general term u_n.

c Hence, find the 12th term as a fraction.

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a $\frac{4}{8} = \frac{1}{2}$ $\quad \frac{2}{4} = \frac{1}{2}$ $\quad \frac{1}{2} = \frac{1}{2}$ $\quad \frac{\frac{1}{2}}{1} = \frac{1}{2}$

Assuming the pattern continues, consecutive terms have a common ratio of $\frac{1}{2}$.

$\therefore$ the sequence is geometric with $u_1 = 8$ and $r = \frac{1}{2}$.

b $u_n = u_1 r^{n-1}$

$\therefore \; u_n = 8\left(\frac{1}{2}\right)^{n-1}$ *or*

$$\begin{aligned} u_n &= 2^3 \times (2^{-1})^{n-1} \\ &= 2^3 \times 2^{-n+1} \\ &= 2^{3+(-n+1)} \\ &= 2^{4-n} \end{aligned}$$

c

$$\begin{aligned} u_{12} &= 8 \times \left(\tfrac{1}{2}\right)^{11} \\ &= \tfrac{1}{256} \end{aligned}$$

Example 8

Self Tutor

$k-1$, $2k$, and $21-k$ are consecutive terms of a geometric sequence. Find k.

Since the terms are geometric, $\quad \dfrac{2k}{k-1} = \dfrac{21-k}{2k}$ $\quad$ {equating rs}

$$\begin{aligned} \therefore \; 4k^2 &= (21-k)(k-1) \\ \therefore \; 4k^2 &= 21k - 21 - k^2 + k \\ \therefore \; 5k^2 - 22k + 21 &= 0 \end{aligned}$$

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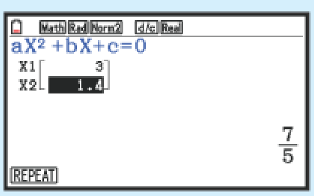

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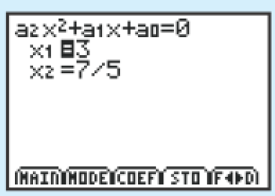

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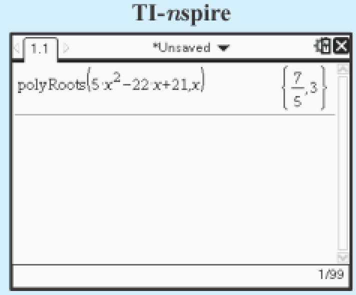

$\therefore \; k = \frac{7}{5}$ or 3 $\quad$ {using technology}

Check: If $k = \frac{7}{5}$ the terms are: $\frac{2}{5}, \frac{14}{5}, \frac{98}{5}$. ✓ $\{r = 7\}$

If $k = 3$ the terms are: $2, 6, 18$. ✓ $\{r = 3\}$

Example 9 — Self Tutor

A geometric sequence has $u_2 = -6$ and $u_5 = 162$. Find its general term.

$$u_2 = u_1 r = -6 \quad .... \ (1)$$

and $$u_5 = u_1 r^4 = 162 \quad .... \ (2)$$

Now $$\frac{u_1 r^4}{u_1 r} = \frac{162}{-6} \qquad \{(2) \div (1)\}$$

$$\therefore \ r^3 = -27$$

$$\therefore \ r = \sqrt[3]{-27}$$

$$\therefore \ r = -3$$

Using (1), $$u_1(-3) = -6$$

$$\therefore \ u_1 = 2$$

Thus $$u_n = 2 \times (-3)^{n-1}.$$

Example 10 — Self Tutor

Find the first term of the sequence $6, 6\sqrt{2}, 12, 12\sqrt{2}, ...$ which exceeds 1400.

The sequence is geometric with $u_1 = 6$ and $r = \sqrt{2}$

$$\therefore \ u_n = 6 \times (\sqrt{2})^{n-1}.$$

We need to find n such that $u_n > 1400$.

Using a graphics calculator with $\text{Y}_1 = 6 \times (\sqrt{2})\text{\^{}}(\text{X} - 1)$, we view a *table of values*:

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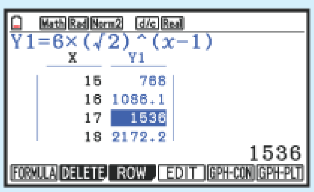

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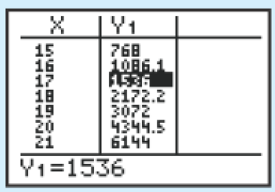

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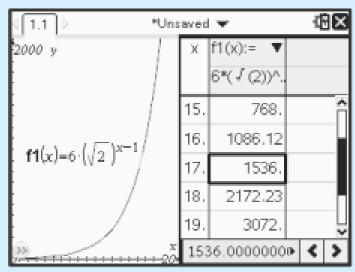

The first term to exceed 1400 is $u_{17} = 1536$.

Example 11 **Self Tutor**

The initial population of rabbits on a farm was 50.
The population increased by 7% each week.

a How many rabbits were present after:

  i 15 weeks **ii** 30 weeks?

b How long would it take for the population to reach 500?

There is a fixed percentage increase each week, so the population forms a geometric sequence.

$u_1 = 50$ and $r = 1.07$

$u_2 = 50 \times 1.07 =$ the population after 1 week

a **i** $u_{n+1} = u_1 \times r^n$

$\therefore \; u_{16} = 50 \times (1.07)^{15}$

≈ 137.95

There were 138 rabbits.

ii $u_{31} = 50 \times (1.07)^{30}$

≈ 380.61

There were 381 rabbits.

b $u_{n+1} = u_1 \times (1.07)^n$ after n weeks

So, we need to find when $50 \times (1.07)^n = 500$.

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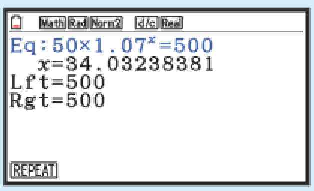

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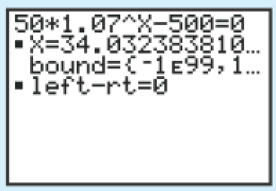

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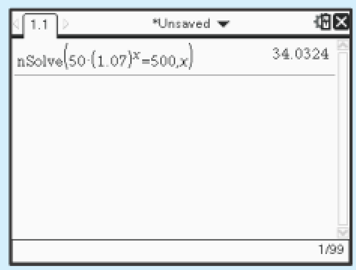

So, it would take approximately 34.0 weeks.