Example 3 **Self Tutor**

Find k given that $3k+1$, k, and -3 are consecutive terms of an arithmetic sequence.

Since the terms are consecutive, $k-(3k+1)=-3-k$ {equating differences}

$$\therefore\ k-3k-1=-3-k$$
$$\therefore\ -2k-1=-3-k$$
$$\therefore\ -1+3=-k+2k$$
$$\therefore\ k=2$$

or Since the middle term is the arithmetic mean of the terms on either side of it,

$$k=\frac{(3k+1)+(-3)}{2}$$
$$\therefore\ 2k=3k-2$$
$$\therefore\ k=2$$

Example 4 Self Tutor

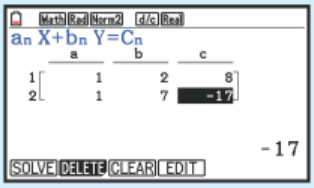
Find the general term u_n for an arithmetic sequence with $u_3 = 8$ and $u_8 = -17$.

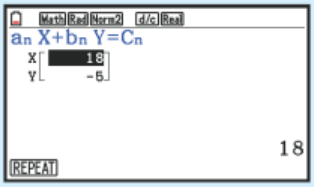
$u_3 = 8 \quad \therefore \quad u_1 + 2d = 8 \quad \ldots. (1) \qquad \{\text{using } u_n = u_1 + (n-1)d\}$

$u_8 = -17 \quad \therefore \quad u_1 + 7d = -17 \quad \ldots. (2)$

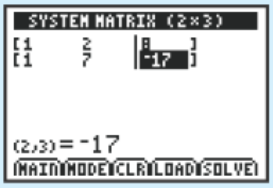
We now solve (1) and (2) simultaneously using technology:

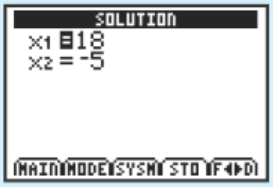
Casio fx-CG20



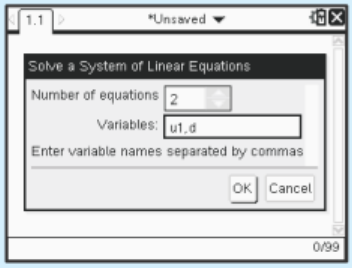


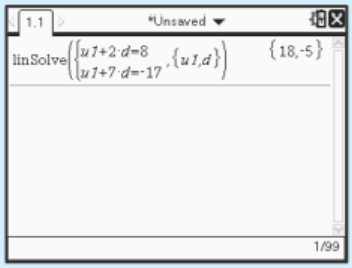
TI-84 Plus





TI-*n*spire





$\therefore \quad u_1 = 18 \quad \text{and} \quad d = -5$

Now $u_n = u_1 + (n-1)d$

$\therefore \quad u_n = 18 - 5(n-1)$

$\therefore \quad u_n = 18 - 5n + 5$

$\therefore \quad u_n = 23 - 5n$

Check:

$u_3 = 23 - 5(3)$ $\quad u_8 = 23 - 5(8)$

$= 23 - 15$ $\quad = 23 - 40$

$= 8$ ✓ $\quad = -17$ ✓

Example 6 **Self Tutor**

Ryan is a cartoonist. His comic strip has just been bought by a newspaper, so he sends them the 28 comic strips he has drawn so far. Each week after the first he mails 3 more comic strips to the newspaper.

a Find the total number of comic strips sent after 1, 2, 3, and 4 weeks.

b Show that the total number of comic strips sent after n weeks forms an arithmetic sequence.

c Find the number of comic strips sent after 15 weeks.

d When does Ryan send his 120th comic strip?

---

a *Week 1*: 28 comic strips
Week 2: $28 + 3 = 31$ comic strips
Week 3: $31 + 3 = 34$ comic strips
Week 4: $34 + 3 = 37$ comic strips

b Every week, Ryan sends 3 comic strips, so the difference between successive weeks is always 3. We have an arithmetic sequence with $u_1 = 28$ and common difference $d = 3$.

c $u_n = u_1 + (n-1)d$
$= 28 + (n-1) \times 3$
$= 25 + 3n$

$\therefore\ u_{15} = 25 + 3 \times 15$
$= 70$

After 15 weeks Ryan has sent 70 comic strips.

d We want to find n such that $u_n = 120$

$\therefore\ 25 + 3n = 120$

$\therefore\ 3n = 95$

$\therefore\ n = 31\frac{2}{3}$

Ryan sends the 120th comic strip in the 32nd week.