Example 17 Self Tutor

The data shows the results of the women's marathon at the 2008 Olympics, for all competitors who finished the race.

- a Construct a cumulative frequency distribution table.
- b Represent the data on a cumulative frequency graph.
- Use your graph to estimate the:
 - i median finishing time
 - ii number of competitors who finished in less than 2 hours 35 minutes
 - iii percentage of competitors who took more than 2 hours 39 minutes to finish
 - iv time taken by a competitor who finished in the top 20% of runners completing the marathon.

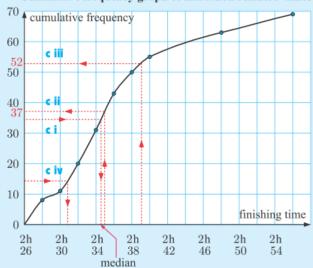
Finishing time t	Frequency	
2 h $26\leqslant t<2$ h 28	8	
$2~\mathrm{h}~28\leqslant t<2~\mathrm{h}~30$	3	
2 h $30\leqslant t<2$ h 32	9	
2 h $32\leqslant t<2$ h 34	11	
2 h $34\leqslant t<2$ h 36	12	
$2~\mathrm{h}~36\leqslant t<2~\mathrm{h}~38$	7	
2 h $38\leqslant t<2$ h 40	5	
$2~\mathrm{h}~40\leqslant t<2~\mathrm{h}~48$	8	
2 h $48\leqslant t<2$ h 56	6	

a	Finishing time t	Frequency	Cumulative frequency
	$2~\mathrm{h}~26\leqslant t<2~\mathrm{h}~28$	8	8
	$2 \text{ h } 28 \leqslant t < 2 \text{ h } 30$	3	11 🕶
	$2 \text{ h } 30 \leqslant t < 2 \text{ h } 32$	9	20
	$2 \text{ h} \ 32 \leqslant t < 2 \text{ h} \ 34$	11	31
	$2~\mathrm{h}~34\leqslant t<2~\mathrm{h}~36$	12	43
	$2~\mathrm{h}~36\leqslant t<2~\mathrm{h}~38$	7	50 -
	$2~\mathrm{h}~38 \leqslant t < 2~\mathrm{h}~40$	5	55
	$2 \text{ h } 40 \leqslant t < 2 \text{ h } 48$	8	63
	$2 \text{ h } 48 \le t < 2 \text{ h } 56$	6	69

8+3=11 competitors completed the marathon in less than 2 hours 30 minutes.

50 competitors completed the marathon in less than 2 hours 38 minutes.

Cumulative frequency graph of marathon runners' times



The cumulative frequency gives a running total of the number of runners finishing by a given time.



- The median is estimated using the 50th percentile. As 50% of 69 is 34.5, we start with the cumulative frequency of 34.5 and find the corresponding time. The median is approximately 2 hours 34.5 min.
 - ii There are approximately 37 competitors who took less than 2 h 35 min to complete the race.
 - iii There are 69-52=17 competitors who took more than 2 hours 39 min. So $\frac{17}{69}\approx 26.4\%$ took more than 2 hours 39 min.
 - iv The time taken is estimated using the 20th percentile. As 20% of 69 is 13.8, we find the time corresponding to a cumulative frequency of approximately 14. The top 20% of competitors took less than 2 hours 31 minutes.