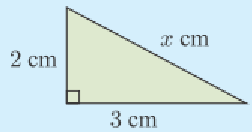
Example 1 **Self Tutor**

Find the length of the hypotenuse in the triangle shown.

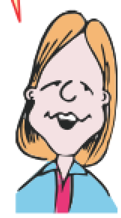


The hypotenuse is opposite the right angle and has length x cm.

$$\therefore \quad x^2 = 3^2 + 2^2 \qquad \{\text{Pythagoras}\}$$
$$\therefore \quad x^2 = 9 + 4$$
$$\therefore \quad x^2 = 13$$
$$\therefore \quad x = \sqrt{13} \qquad \{\text{as } x > 0\}$$

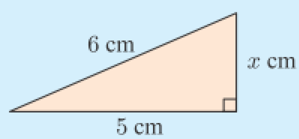
So, the hypotenuse is $\sqrt{13}$ cm long.

If $x^2 = k$, then $x = \pm\sqrt{k}$. We reject $-\sqrt{k}$ as lengths must be positive!



Example 2 **Self Tutor**

Find the length of the third side of the given triangle.



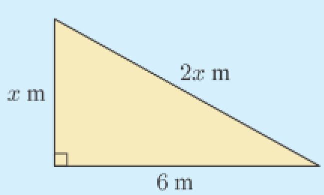
The hypotenuse has length 6 cm.

$$\therefore \quad x^2 + 5^2 = 6^2 \qquad \{\text{Pythagoras}\}$$
$$\therefore \quad x^2 + 25 = 36$$
$$\therefore \quad x^2 = 11$$
$$\therefore \quad x = \sqrt{11} \qquad \{\text{as } x > 0\}$$

So, the third side is $\sqrt{11}$ cm long.

Example 5 **Self Tutor**

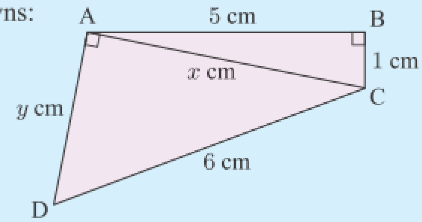
Find the value of x:



$$(2x)^2 = x^2 + 6^2 \qquad \{\text{Pythagoras}\}$$
$$\therefore \quad 4x^2 = x^2 + 36$$
$$\therefore \quad 3x^2 = 36$$
$$\therefore \quad x^2 = 12$$
$$\therefore \quad x = \pm\sqrt{12}$$
$$\therefore \quad x = \sqrt{12} \qquad \{\text{as } x > 0\}$$

Example 6 Self Tutor

Find the value of any unknowns:



In $\triangle$ABC, the hypotenuse is x cm long.

$\therefore \; x^2 = 5^2 + 1^2$ {Pythagoras}

$\therefore \; x^2 = 26$

$\therefore \; x = \sqrt{26}$ {as $x > 0$}

In $\triangle$ACD, the hypotenuse is 6 cm long.

$\therefore \; y^2 + (\sqrt{26})^2 = 6^2$ {Pythagoras}

$\therefore \; y^2 + 26 = 36$

$\therefore \; y^2 = 10$

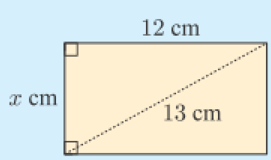
$\therefore \; y = \sqrt{10}$ {as $y > 0$}

Example 7 Self Tutor

The longer side of a rectangle is 12 cm and its diagonal is 13 cm. Find:

a the length of the shorter side **b** the area of the rectangle.

a



Let the shorter side be x cm.

$\therefore \; x^2 + 12^2 = 13^2$ {Pythagoras}

$\therefore \; x^2 + 144 = 169$

$\therefore \; x^2 = 25$

$\therefore \; x = 5$ {as $x > 0$}

So, the shorter side is 5 cm long.

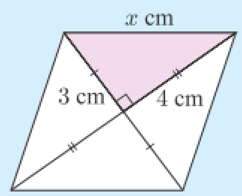
b Area = length $\times$ width

$= 12 \times 5$

$= 60 \text{ cm}^2$

Example 8 — Self Tutor

A rhombus has diagonals of length 6 cm and 8 cm. Find the length of its sides.



The diagonals of a rhombus *bisect at right angles*.

Let each side be x cm long.

$\therefore\ x^2 = 3^2 + 4^2$ {Pythagoras}

$\therefore\ x^2 = 9 + 16$

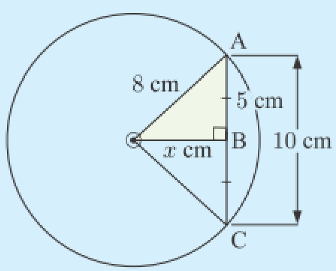
$\therefore\ x^2 = 25$

$\therefore\ x = 5$ {as $x > 0$}

So, the sides are 5 cm long.

Example 10 — Self Tutor

A circle of radius 8 cm has a chord of length 10 cm.
Find the shortest distance from the centre of the circle to the chord.



The shortest distance is the 'perpendicular distance'. The line drawn from the centre of a circle, at right angles to a chord, bisects the chord, so

AB = BC = 5 cm

In $\triangle$AOB, $5^2 + x^2 = 8^2$ {Pythagoras}

$\therefore\ x^2 = 64 - 25 = 39$

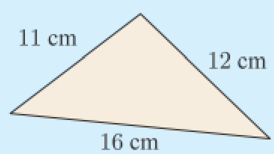
$\therefore\ x = \sqrt{39}$ {as $x > 0$}

$\therefore\ x \approx 6.24$

So, the shortest distance is 6.24 cm.

Example 11 — Self Tutor

The dimensions marked on this triangle are correct, but the triangle is not drawn to scale. Is it a right angled triangle?



The two shorter sides have lengths 11 cm and 12 cm.

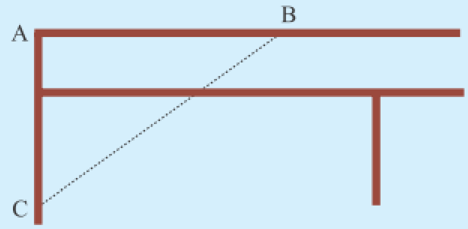
Now $11^2 + 12^2 = 121 + 144 = 265$

whereas $16^2 = 256$

Since $11^2 + 12^2 \neq 16^2$, the triangle is not right angled.

Example 13 **Self Tutor**

Bjorn suspects that the corner A of a tennis court is not a right angle.
With a measuring tape he finds that $AB = 3.72$ m, $BC = 4.56$ m, and $AC = 2.64$ m.
Is Bjorn's suspicion correct?



$$BC^2 = 4.56^2 \approx 20.8$$

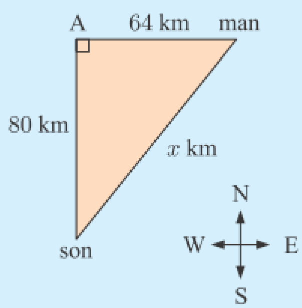
and $AB^2 + AC^2 = 3.72^2 + 2.64^2 \approx 20.8$ {to 3 significant figures}

Within the limitations of accuracy of the measurements, the angle at A is a right angle.

Example 15 **Self Tutor**

A man and his son leave point A at the same time. The man cycles due east at 16 km h^{-1}. His son cycles due south at 20 km h^{-1}.

How far apart are they after 4 hours?



After 4 hours the man has travelled $4 \times 16 = 64$ km and his son has travelled $4 \times 20 = 80$ km.

Thus $x^2 = 64^2 + 80^2$ {Pythagoras}

$\therefore\ x^2 = 4096 + 6400$

$\therefore\ x^2 = 10\,496$

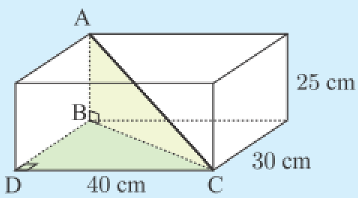
$\therefore\ x = \sqrt{10\,496}$ {as $x > 0$}

$\therefore\ x \approx 102.4$

They are about 102 km apart after 4 hours.

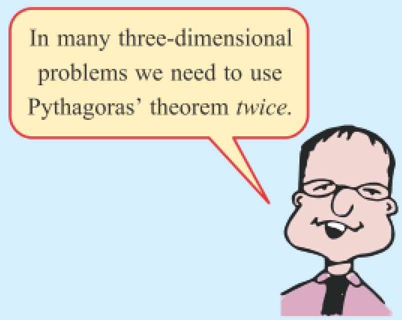
Example 17 — Self Tutor

Skyways Airlines has the policy that passengers cannot carry on luggage with diagonal measurement of more than 56 cm. Katie's bag is 40 cm × 30 cm × 25 cm. Is she allowed to carry it on board the plane?



We first consider the distance BC across the base.

By Pythagoras, $BC^2 = 40^2 + 30^2$



Now triangle ABC is right angled at B.

$\therefore\ AC^2 = AB^2 + BC^2$ {Pythagoras}

$\therefore\ AC^2 = 25^2 + 40^2 + 30^2$

$\therefore\ AC = \sqrt{(25^2 + 40^2 + 30^2)}$ {as $AC > 0$}

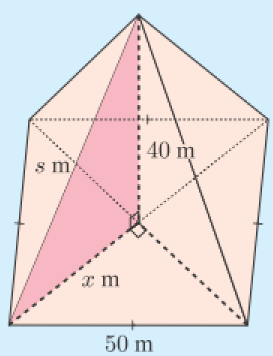
$\therefore\ AC \approx 55.9$ cm

So, Katie is allowed to carry her bag on the plane.

Example 18 — Self Tutor

A pyramid of height 40 m has a square base with edges 50 m long.

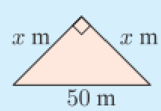
Determine the length of the slant edges.



Let a slant edge have length s m.

Let half a diagonal have length x m.

Using

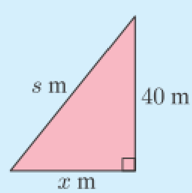


$x^2 + x^2 = 50^2$ {Pythagoras}

$\therefore\ 2x^2 = 2500$

$\therefore\ x^2 = 1250$

Using



$s^2 = x^2 + 40^2$ {Pythagoras}

$\therefore\ s^2 = 1250 + 1600$

$\therefore\ s^2 = 2850$

$\therefore\ s = \sqrt{2850}$ {as $s > 0$}

$\therefore\ s \approx 53.4$

So, each slant edge is about 53.4 m long.